Fall, 2023 **06-606**

Nonlinear Optimization in Process Systems Engineering

Assignment #2 **Due: 9/21/23**

1. Given the system of equations

$f_1 = 2x_1^2 + x_2^2 - 6 = 0$
$f_2 = x_1 + 2x_2 - 3.5 = 0$

a. Solve with Newton's method (unit step size) using as a starting point
$x_1 = 2.0,\ x_2 = 1.0$

b. Using as a starting point $x_1 = x_2 = 0$, solve the system with Newton's method with the Armijo line search. What problems are likely to occur from this starting point and what must you do to get the method to converge to a tolerance of $f^T f < 10^{-8}$?

2. Given is the system of linear equations in n variables x

$$f(x) = b + Ax = 0$$

where A is a non-singular matrix. Show the convergence properties of Newton's method. What properties does Broyden's method have on such a system (some literature search is required here)?

3. Reformulate the following equations so they do not have poles. Why is this necessary?

$f_1 = \exp(x/(y^2 - 6))/z + 6 = 0$
$f_2 = 6 \ln(1/z^2)/t + 6 = 0$

4. For the system of equations in problem 1, apply Broyden's method to find the solution with the starting point given in 1a Use the update formula for the inverse of the Jacobian and estimate the initial Jacobian J^0 by using a finite difference perturbation of 10^{-3}. The criterion for convergence is the same as in problem 1.

5. Research the MINPACK or Numpy library and describe which subroutines would be appropriate to solve problems 1a,b and 4.

6. Derive the quadratic rule for stepsize adjustment (α_q) that is used in step d. of the Armijo linesearch.

7. For Broyden's method:

a) Assume that B^0 is symmetric. Derive a symmetric Broyden updating formula of the form: $B^{k+1} = B^k + u\,u^T$ which satisfies the secant relation.

b) Derive the analogous symmetric inverse update formula without using B^k in the final formula.

c) Verify that $B^{k+1} = B^k + (y - B^k s)c^T/c^T s$ satisfies the secant relation for an arbitrary vector c.