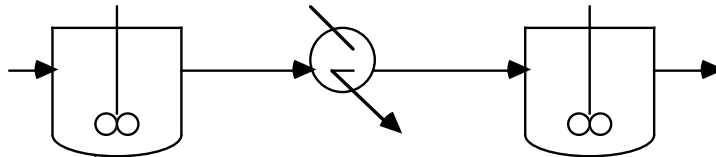


1. Using Taylor series expansion and the chain rule for partial differentiation for $dy/dx = f(x, y)$, derive the implicit 2nd order Gauss formula

$$y_{n+1} = y_n + k_1$$

$$k_1 = h f(x_n + h/2, y_n + k_1/2)$$

2. The reaction $A \rightarrow B$ takes place at steady state in two isothermal CSTR reactors in series as shown below in the figure, with a constant flowrate of $100 \text{ ft}^3/\text{min}$. Obtain the concentrations C_{A1} , C_{A2} as a function of time given that the inlet concentration C_{A0} is perturbed from 1.5 to 2.0 moles/ft^3 . Integrate for the first ten minutes using your favorite method. Plot your results and compare them with the analytical solution.



$$V_1 = 500 \text{ ft}^3$$

$$k_1 = 0.05 \text{ min}^{-1}$$

$$V_2 = 20 \text{ ft}^3$$

$$k_2 = 38 \text{ min}^{-1}$$

3. Show that the trapezoidal rule: $y_{n+1} = y_n + h/2 \{f(x_n, y_n) + f(x_{n+1}, y_{n+1})\}$ also corresponds to a 2nd order implicit Runge-Kutta method, and obtain its coefficients in the Butcher block matrix.

4. Show that for any v , the following Runge-Kutta method, is consistent of order 2.

$$y_{n+1} = y_n + 1/2 (k_2 + k_3),$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + vh, y_n + v k_1)$$

$$k_3 = hf(x_n + (1-v)h, y_n + (1-v) k_1)$$

5. Investigate the numerical stability of the following methods for $y' = \lambda y$, $\lambda < 0$

i) $y_{n+1} = y_n + h/12[5f_{n+1} + 8f_n - f_{n-1}]$ (3rd Order Adams-Moulton Corrector)

ii) $y_{n+1} = y_{n-1} + h/3[f_{n+1} + 4f_n + f_{n-1}]$ (Milne-Simpson Formula)

iii) $y_{n+1} = y_{n-1} + h/2[f_n + 3f_{n-1}]$

6. The equation $y' = -Ay + B$ has a general solution $y(x) = c e^{-Ax} + B/A$ where c is an arbitrary constant and thus $y(x) \rightarrow B/A$ as x goes to infinity. If Euler's method is applied to this equation, show that $y_n \rightarrow B/A$ as n goes to infinity only if $h < 2/A$.

7. Consider the system

$$y_1' = -0.1 y_1 - 49.9 y_2 \quad y_1(0) = 3$$

$$y_2' = -50 y_2 \quad y_2(0) = 1.5$$

$$y_3' = 70 y_2 - 120 y_3 \quad y_3(0) = 3$$

- Calculate the stiffness ratio and the eigenvalues for this system
- Find the analytic solution.
- Find the maximum value for which h is stable for a fourth order explicit Runge-Kutta method (see Carnahan and Wilkes paper on website for the stability plot.)