

## Homework Assignment 6

1. While searching for the minimum of

$$f(x) = [x_1^2 + (x_2 + 1)^2][x_1^2 + (x_2 - 1)^2]$$

the algorithm terminates at the following points:

- a)  $x^{(1)} = [0, 0]^T$
- b)  $x^{(2)} = [0, 1]^T$
- c)  $x^{(3)} = [0, -1]^T$
- d)  $x^{(4)} = [1, 1]^T$

Classify each point.

2. Consider the quadratic function with the parameter M:

$$f(x) = 3x_1 + x_2 + 2x_3 + 4x_1^2 + 3x_2^2 + 2x_3^2 + (M-2)x_1x_2 + 2x_2x_3$$

For  $M = 0$  find all stationary points. Are they optimal? Find the path of optimal solutions as M increases from zero.

3. In Powell damping, the BFGS update is modified if  $s^T y$  is not sufficiently positive by defining  $\bar{y} = \theta y + (1 - \theta)B^k s$  and substituting for  $y$  in the BFGS formula.

a) Show that  $\theta$  can be found by solving the one-dimensional linear program:

$$\max \theta \text{ s. t. } \theta s^T y + (1 - \theta)s^T B^k s \geq 0.2 s^T B^k s, \theta \in [0, 1]$$

b) If  $s^T y \geq 0.2 s^T B^k s$  show that Powell damping corresponds to a normal BFGS update.

c) If  $s^T y \rightarrow -\infty$ , show that Powell damping corresponds to skipping the BFGS update.

4. Show that if  $B^k$  is positive definite  $\cos \theta^k > 1/\kappa(B^k)$  where  $\kappa(B^k)$  is the condition number of  $B^k$ , based on the 2-norm.

5. Derive a stepsize rule for  $\alpha$  for the Armijo line search that minimizes the quadratic interpolant from the Armijo inequality.

6. Consider the convex problem:

$$\min x_1 \text{ s. t. } x_2 \leq 0, x_2 - x_1^2 \geq 0$$

Show that this problem does not satisfy LICQ and does not satisfy the KKT conditions at its optimum solution.

7. Consider the convex problem

$$\min f(x) \text{ s.t. } g(x) \leq 0$$

and the equivalent problem

$$\min f(x) \text{ s.t. } g(x) + s = 0, s \geq 0.$$

- a) Show that the KKT conditions of the two problems are equivalent.
- b) If the second problem has a local solution. Show that this is also a global solution.