06-606 **Computational Methods** **Spring 2020**

Due: 4/9/20

1. While searching for the minimum of

$$f(x) = [x_1^2 + (x_2+ 1)^2][x_1^2 + (x_2 - 1)^2]$$

the algorithm terminates at the following points:

a) $x^{(1)} = [0,0]^T$
b) $x^{(2)} = [0,1]^T$
c) $x^{(3)} = [0,-1]^T$
d) $x^{(4)} = [1,1]^T$

Classify each point.

2. Consider the quadratic function with the parameter M:

$$f(x) = 3x_1 + x_2 + 2x_3+4x_1^2 + 3x_2^2 + 2x_3^2+ (M-2)\, x_1 x_2 +2x_2x_3$$

For M = 0 find all stationary points. Are they optimal? Find the path of <u>optimal</u> solutions as M increases from zero.

3. In Powell damping, the BFGS update is modified if s^Ty is not sufficiently positive by defining $\bar{y} = \theta y + (1-\theta)B^k s$ and substituting for y in the BFGS formula.

a) Show that θ can by found by solving the one-dimensional linear program:

$$\max \theta \; s.t. \; \theta s^T y + (1-\theta) s^T B^k s \geq 0.2\; s^T B^k s, \theta \in [0, 1]$$

b) If $s^Ty \geq 0.2$ s^TBs show that Powell damping corresponds to a normal BFGS update.

c) If s^Ty —>$-\infty$, show that Powell damping corresponds to skipping the BFGS update.

4. Show that if B^k is positive definite $\cos\theta > 1/\kappa(B^k)$ where $\kappa(B^k)$ is the condition number of B^k, based on the 2-norm.

5. Derive a stepsize rule for α for the Armijo line search that minimizes the quadratic interpolant from the Armijo inequality.

6. Consider the convex problem:

$$\min x_1 \; s.t. \; x_2 \leq 0, x_2 - x_1^2 \geq 0$$

Show that this problem does not satisfy LICQ and does not satisfy the KKT conditions at its optimum solution.

7. Consider the convex problem

$$\min f(x)\ \ s.t.\ g(x) \leq 0$$

and the equivalent problem

$$\min f(x)\ \ s.t.\ g(x) + s = \ 0, s \geq 0.$$

a) Show that the KKT conditions of the two problems are equivalent.
b) If the second problem has a local solution. Show that this is also a global solution.