

1. Consider the problem:

$$\begin{aligned} & \text{Min } F(z(t_f)) \\ \text{s.t. } & z' = f(z, u), z(0) = z_0 \\ & a \leq u(t) \leq b \end{aligned}$$

- write the Euler-Lagrange conditions for this problem
- formulate this problem as an NLP using orthogonal collocation on finite elements
- write the Kuhn Tucker conditions for this problem
- using integration by parts for the integral in $(\lambda^T z')$ in a) and c) show that the Kuhn Tucker conditions are a discrete analog of the Euler-Lagrange conditions. (Hint: Cuthrell and Biegler, "Simultaneous Optimization and Solution Methods for Batch Reactor Control Profiles," Computers and Chemical Engineering 13, 1/2, p. 49 (1989))

2. Consider the reactor optimal control problem:

$$\begin{aligned} & \text{Max } c_2(1.0) \\ \text{s.t. } & c_1' = -k_1(T) c_1^2 \\ & c_2' = k_1(T) c_1^2 - k_2(T) c_2 \\ & c_1(0) = 1, c_2(0) = 0 \end{aligned}$$

where $k_1 = 4000 \exp(-2500/T)$, $k_2 = 62000 \exp(-5000/T)$ and T lies between 298 and 398 K

- formulate the optimality conditions for this problem as a boundary value problem.
- using either COLDAE or GAMS, solve this problem using the boundary value formulation and orthogonal collocation on finite elements.

3. Consider the optimal control problem:

$$\begin{aligned} & \text{Min } x_3(1.0) \\ \text{s.t. } & x_1' = x_2 \\ & x_2' = -x_2 + u \\ & x_3' = x_1^2 + x_2^2 + 0.005 u^2 \\ & \text{with } x_1(0) = 0, x_2(0) = -1, x_3(0) = 0. \end{aligned}$$

a) Solve this problem in GAMS using orthogonal collocation on finite elements

b) Solve this problem in GAMS using orthogonal collocation on finite elements
With the path constraint: $x_2(t) \leq 8(t-0.5)^2 - 0.5$