06-606 **Computational Methods** **Spring 2020**

Due: 5/1/20

1. Consider the problem:

$$\text{Min } F(z(t_f))$$
$$\text{s.t. } z' = f(z, u),\ z(0) = z_0$$
$$a \le u(t) \le b$$

a) write the Euler-Lagrange conditions for this problem
b) formulate this problem as an NLP using orthogonal collocation on finite elements
c) write the Kuhn Tucker conditions for this problem
d) using integration by parts for the integral in ($\lambda^T z'$) in a) and c) show that the Kuhn Tucker conditions are a discrete analog of the Euler-Lagrange conditions. (Hint: Cuthrell and Biegler, "Simultaneous Optimization and Solution Methods for Batch Reactor Control Profiles," Computers and Chemical Engineering 13, 1/2, p. 49 (1989))

2. Consider the reactor optimal control problem:

Max $c_2(1.0)$

s.t. $c_1' = -k_1(T)\, c_1^2$
$c_2' = k_1(T)\, c_1^2 - k_2(T)\, c_2$
$c_1(0) = 1,\ c_2(0) = 0$

where $k_1 = 4000 \exp(-2500/T)$, $k_2 = 62000 \exp(-5000/T)$ and T lies between 298 and 398 K
a) formulate the optimality conditions for this problem as a boundary value problem.
b) using either COLDAE or GAMS, solve this problem using the boundary value formulation and orthogonal collocation on finite elements.

3. Consider the optimal control problem:
Min $x_3(1.0)$

s.t. $x_1' = x_2$
$x_2' = -x_2 + u$
$x_3' = x_1^2 + x_2^2 + 0.005\, u^2$

with $x_1(0) = 0$, $x_2(0) = -1$, $x_3(0) = 0$.

a) Solve this problem in GAMS using orthogonal collocation on finite elements

b) Solve this problem in GAMS using orthogonal collocation on finite elements
With the path constraint: $x_2(t) \le 8(t-0.5)^2 - 0.5$