Computational Methods - Exam II

06-606 3/18/93

This 90 minute exam consists of four unequally weighted parts. Please budget your time carefully. You may consult your class notes, textbooks and homework sets to solve these problems.

1. Consider the binary flash-condenser system given below. The mass balance equations can be written as:

$$M_f' = L - V$$
$$(M_f x_f)' = L\, x_c - V\, y$$
$$(M_c)' = V - (L + D)$$
$$(M_c x_c)' = V\, y - (L+D)\, x_c$$
$$y - K\, x_f = 0, \qquad M_f = f(t)$$
$$L = V/2, \qquad D = g(t)$$

a) Formulate this problem as a semi-explicit DAE system. What is the index of this system? (10 points)
b) Reformulate this problem to an index 1 system. (10 points)

2. Consider the following DAE system:

$$x_1' = x_2; \quad x_2' = x_3; \quad x_3' = x_4; \quad x_1 = f(t)$$

a) What is the index of this DAE system? (10 points)
b) Stabilize this DAE system with the index 2 formulation. (10 points)
c) Using an index 1 formulation, find a consistent initialization for this problem. (10 points)

3) Consider the two point boundary point problem,

$$u'' = a\, u' + b\, u + f(t); \qquad u(0) = \alpha,\ u(1)' = \beta.$$

Modify the error and stability analysis given in class to consider a right boundary condition of the form: $(u_{N-1} - u_N)/h = \beta$. Set up but do not solve the difference equations for e_i. (15 points)

4) For the system

$$y'' + y' + y = 0; \qquad y(0) = -1, \quad y(0) = 1$$

a) describe a detailed shooting method for first order ODE's. Do not solve. (10 points)
b) using a power series basis apply two point orthogonal collocation and write the equations. What are the collocation points? (15 points)
c) apply an equivalent order Galerkin method and write the equations. (10 points)