06-720 Advanced Process Systems Engineering Spring 2011
Homework 2 Due: 1/26/11

1. Consider the convex problem:

$$\min x_1$$
$$\text{s.t. } x_2 \le 0;$$
$$x_1^2 - x_2 \le 0$$

Show that this problem does not satisfy LICQ and does not satisfy the KKT conditions at the optimum.

2. Quasi-Newton Methods

In the derivation of the Broyden update, the following convex equality constrained problem is solved:

$$\min \|J^+ - J\|_F^2$$
$$s.t.\quad J^+ s = y$$

Using the definition of the Frobenius norm from Section 2.2.1, apply the optimality conditions to the elements of J^+ and derive the Broyden update: $J^+ = J + \frac{(y-Js)s^T}{s^T s}$.

3. NLP Reformulation

A widely used trick is to convert (4.1) to an equality constrained problem by adding new variables z_j to each inequality constraint to form: $g_j(x) - (z_j)^2 = 0$. Compare the KKT conditions for the converted problem with (4.1). Discuss any differences between these conditions as well as the implications of using the converted form within an NLP solver.

4. Find the solution to: Min x_1 s.t. $x_2 \le x_1^3$, $-x_1^3 \le x_2$
and show that it does not satisfy KKT conditions. Explain why.

5. Consider the NLP problem:

$$\text{Min } x_1 + (3x_2 - 1)^2$$
$$\text{s.t. } 2x_1 + x_2 - x_3 = 0$$
$$x_1, x_2, x_3 \ge 0$$

a) Write the first order KKT conditions and find the solution and multipliers for this problem.

b) At the solution of a) are the sufficient second order KKT conditions satisfied?