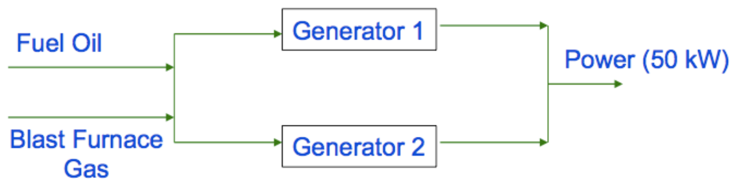
06-720 **Advanced Process Systems Engineering** **Spring, 2011**

. **Homework 4** **Due: 2/9/11**

1. A two-boiler turbine-generator combination shown below



is used to produce a power output of 50 MW using a combination of fuel oil or blast furnace gas (BFG). However, to run these generators, BFG is limited to 12 units per hour and additional fuel oil needs to be purchased. To determine the amount of fuel needed, a curve fit has been done so that generator power (x) and fuel required (f) is given by $f = a_0 + a_1 x + a_2 x^2$. Coefficients for this correlation are given below.

generator	fuel	a_0	a_1	a_2
1	oil	1.4609	0.15186	0.00145
1	gas	1.5742	0.16310	0.001358
2	oil	0.8008	0.20310	0.000916
2	gas	0.7266	0.22560	0.000778

Assume that the generators each produce power between 15 and 35.

(a) Based on the above information, formulate and solve an optimization problem to minimize the amount of fuel oil purchased using the GAMS solver.

(b) Does the above problem satisfy sufficient second order conditions? What is the reduced Hessian?

(c) If fuel oil is limited to 10 units per hour and BFG is purchased, What is the minimum BFG for 50 MW?

(d) For the above problem, what are the sensitivies of the optimum to a) increasing the power output, b) changing the limits of the generators?

2. Rederive the interior point method developed for $\min f(x), s.t.\ c(x) = 0, x \geq 0$ to the double-bounded NLP, $\min f(x), s.t.\ c(x) = 0, x_l \leq x \leq x_u$.

(a) Extend the derivation of the primal-dual equations for $\min f(x), s.t.\ c(x) = 0, x \geq 0$ to the double-bounded NLP.

(b) Derive the resulting Newton step for these equations and the associate KKT matrix.