

1. For the system  $xy = 1, x \in [1/2, 2], y \in [1/2, 2]$ :
  - (a) Plot the McCormick relaxation for this problem.
  - (b) If the region is partitioned into four with  $x \leq 1, x \geq 1$  and  $y \leq 1, y \geq 1$ , plot the resulting regions.
2. Solve the problem  $\max x + y, s.t. xy \leq 4, x \in [0, 6], y \in [0, 4]$ .
3. Consider the following NLP:

$$\begin{aligned}
 \min \quad & x_1 - x_2 - x_3 - x_1x_3 + x_1x_4 + x_2x_3 - x_2x_4 \\
 \text{s.t.} \quad & x_1 + 4x_2 \leq 8 \\
 & 4x_1 + x_2 \leq 12 \\
 & 3x_1 + 4x_2 \leq 12 \\
 & 2x_3 + x_4 \leq 8 \\
 & x_3 + 2x_4 \leq 8 \\
 & x_3 + x_4 \leq 5 \\
 & 0 \leq x_1, x_2, x_3, x_4 \leq 10
 \end{aligned}$$

- (a) Apply McCormick convex envelopes and develop the LP lower bounding problems. Solve the problem to a global solution.
  - (b) Verify the solution to this problem by solving it with BARON.
4. Consider the integer programming problem:

$$\begin{aligned}
 \max \quad & 1.2y_1 + y_2 \\
 \text{s.t.} \quad & y_1 + y_2 \leq 1 \\
 & 1.2y_1 + 0.5y_2 \leq 1 \\
 & y_1, y_2 = \{0, 1\}
 \end{aligned}$$

- (a) Determine from inspection the solution of the relaxed problem.
  - (b) Enumerate the four 0-1 combinations in your plot to find the optimal solution.
  - (c) Solve the relaxed LP problem by hand and derive Gomory cuts based on the LP relaxation. Verify that they cut-off the relaxed LP solution. *This part is optional and will not be graded. See the solution when posted.*

- (d) Solve the above problem with the branch and bound method by enumerating the nodes in the tree and solving the LP subproblems with GAMS.
5. A company is considering to produce a chemical  $C$  which can be manufactured with either process II or process III, both of which use as raw material chemical  $B$ .  $B$  can be purchased from another company or else manufactured with process I which uses  $A$  as a raw material. Consider the two following cases:
1. Maximum demand of  $C$  is 10 tons/hr with a selling price of \$1800/ton.
  2. Maximum demand of  $C$  is 15 tons/hr; the selling price for the first 10 ton/hr is \$1800/ton, and \$1500/ton for the excess.

Investment and Operating Costs:

	Fixed (\$/hr)	Variable(\$/ton raw mat)
Process I	1000	250
Process II	1500	400
Process III	2000	550

Prices:

A: \$500/ton  
 B: \$950/ton

Conversions:

Process I: 90% of A to B  
 Process II: 82% of B to C  
 Process III: 95% of B to C

Maximum supply of A: 16 tons/hr

Given the specifications above, formulate an MILP model and solve it with GAMS to decide:

- (a) Which process to build (II and III are exclusive)?
- (b) How to obtain chemical B?
- (c) How much should be produced of product C? The objective is to maximize profit.