06-720 **Advanced Process Systems Engineering** **Spring, 2011**

. **Homework 8** **Due: 4/11/11**

1. Use MATLAB to show that the solution profiles for the example:

$$\begin{aligned} \min \quad & -b(1) \\ s.t. \quad \frac{da}{dt} &= -a(t)(u(t) + 0.5u(t)^2) \\ \frac{db}{dt} &= a(t)u(t) \\ a(0) &= 1, \quad b(0) = 0, \quad u(t) \in [0, 5] \end{aligned}$$

satisfy the optimality conditions.

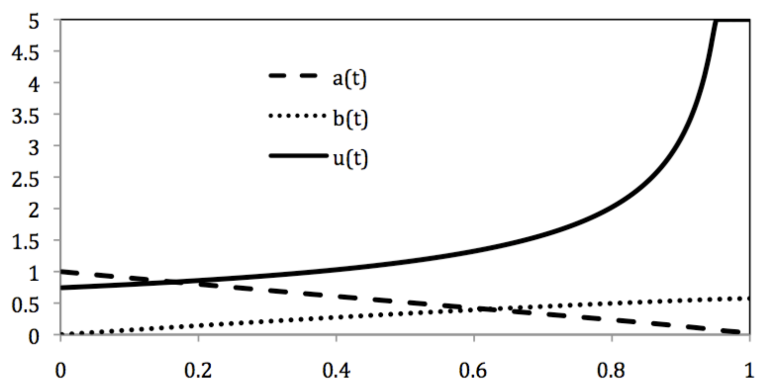

Figure 1: State and Control Profiles for Batch Reactor: $A \to B, A \to C$.

2. For the singular control problem given by:

$$\min \quad \Phi(z(t)), \quad s.t. \ \frac{dz(t)}{dt} = f_1(z) + f_2(z)u(t), z(0) = z_0, u(t) \in [u_L, u_U].$$

Show that $q \geq 2$ in $\frac{d^q H_u(t)}{dt^q} = \varphi(z, u) = 0$.

3. For the problem below, derive the two point boundary value problem and show the relationship of $u(t)$ to the state and adjoint variables.

$$\begin{aligned} \min \quad & z_1(1)^2 + z_2(1)^2 \\ s.t. \quad & \frac{dz_1(t)}{dt} = -2z_2, \ z_1(0) = 1 \\ & \frac{dz_2(t)}{dt} = z_1 u(t), \ z_2(0) = 1 \end{aligned}$$

4. Solve the Catalyst Example given below for the case where the first reaction is irreversible ($k_2 = 0$). Show that the solution is bang-bang.

$$
\begin{aligned}
&\min \quad && a(t_f) + b(t_f) - a_0 \\
s.t. \quad &\frac{da(t)}{dt} &=\;& -u(k_1 a(t) - k_2 b(t)) \\
&\frac{db(t)}{dt} &=\;& u(k_1 a(t) - k_2 b(t)) - (1-u)k_3 b(t) \\
&a(0) &=\;& a_0, \quad b(0) = 0, \quad u(t) \in [0, 1].
\end{aligned}
$$