**06-720 Advanced Process Systems Engineering Spring 2011**
**Homework 1 Due: 1/19/11**

1. Consider the minimization problem: min |x| + |y| s.t. $x^2 + y^2 = 1$. By introducing binary decision variables to handle the absolute value terms, reformulate this problem as a mixed integer programming problem of the form.

2. Examine whether the following functions are convex or not.

$x^2 + ax + b;\ x \in R$
$x^3;\ x \in R$
$x^4;\ x \in R$
$\log(x);\ x \in (0; 1]$

3. Consider the quadratic function:

$$f(x) = 3x_1 + x_2 + 2x_3 + 4x_1^2 + 3x_2^2 + 2x_3^2 + (M-2)x_1x_2 + 2x_2x_3$$

For M= 0 find the eigenvalues and eigenvectors and any stationary points. Are the stationary points local optima? global optima? Find the path of optimal solutions as M increases.

4. Prove that a Hessian matrix which is positive definite has positive eigenvalues.

5. Show that if $B^k$ is positive definite, then $\cos \theta^k > 1/\kappa(B^k)$ where $\kappa(B^k)$ is the condition number of $B^k$, based on the 2-norm.

6. Derive a stepsize rule for the Armijo linesearch that minimizes the quadratic interpolant from the Armijo inequality.