06-720 **Advanced Process Systems Engineering** Spring, 2011

Homework 6 Due: 3/16/11

1. Consider three common rules for the synthesis of distillation sequences.

$$P_1 \wedge \neg P_2 \implies \neg P_3$$
$$\neg P_1 \wedge P_4 \implies P_5$$
$$P_2 \implies P_3$$

where

P_1 = lowest concentration component
P_2 = most volatile component
P_3 = remove component from top of column
P_4 = easy component to separate
P_5 = remove component first

(a) Write these logical expressions as English sentences.

(b) Rewrite rules in conjunctive normal form and write as constraints with binary variables.

2. Using GAMS solve the following MINLP problem step by step with

a) Generalized Benders decomposition
b) Outer-approximation method
c) Extended cutting plane

Also verify your answer with GAMS/DICOPT.

$$\min f = y_1 + 1.5y_2 + 0.5y_3 + x_1^2 + x_2^2$$

s.t.

$$(x_1 - 2)^2 - x_2 \leq 0$$
$$x_1 - 2y_1 \geq 0$$
$$x_1 - x_2 - 4(1-y_2) \leq 0$$
$$x_1 - (1 - y_1) \geq 0$$
$$x_2 - y_2 \geq 0$$
$$x_1 + x_2 \geq 3y_3$$
$$y_1 + y_2 + y_3 \geq 1$$
$$0 \leq x_1 \leq 4, \quad 0 \leq x_2 \leq 4$$
$$y_1, y_2, y_3 = 0, 1$$

Starting point $y_1 = y_2 = y_3 = 1$

$x_1 = x_2 = x_3 = 0$ for extended cutting plane.

3. For the Generalized Disjunctive Program given below,

a) Reformulate it as an MINLP using the convex hull formulation for the disjunction
b) Reformulate it as a big-M MINLP (M=50)
c) Solve both reformulations and compare their relaxations.

$$\min Z = c + (x_1 - 3)^2 + (x_2 - 2)^2$$
$$st$$
$$\begin{bmatrix} Y_1 \\ x_1^2 + x_2^2 \le 1 \\ c = 2 \end{bmatrix} \vee \begin{bmatrix} Y_2 \\ (x_1 - 4)^2 + (x_2 - 1)^2 \le 1 \\ c = 1 \end{bmatrix} \vee \begin{bmatrix} Y_3 \\ (x_1 - 2)^2 + (x_2 - 4)^2 \le 1 \\ c = 3 \end{bmatrix}$$
$$0 \le x_1 \le 8, 0 \le x_2 \le 8, \quad Y_j = true, false, j = 1,2,3$$

4. It is proposed to manufacture a chemical C with a process I that uses raw material B. B can either be purchased or manufactured with either of two processes, II or III, which use chemical A as a raw material. In order to decide the optimal selection of processes and levels of production that maximize profit formulate the MINLP problem and solve with the augmented penalty/outer-approximation/equality-relaxation algorithm in DICOPT++.

Data:

Conversion:	Process I	C = 0.9B	
	Process II	B = ln(1 + A)	Maximum capacity: 5 ton prod/hr
	Process III	B = 1.2 ln (1 + A)	
		(A, B, C, in ton/hr)	

Prices:	A $ 1,800/ton
	B $ 7,000/ton
	C $13,000/ton (maximum demand: 1 ton/hr)

Investment cost

	Fixed (10^3\$/hr)	Variable (10^3\$/ton product)
Process I	3.5	2
Process II	1	1
Process III	1.5	1.2

Note: Minimize negative of profit.