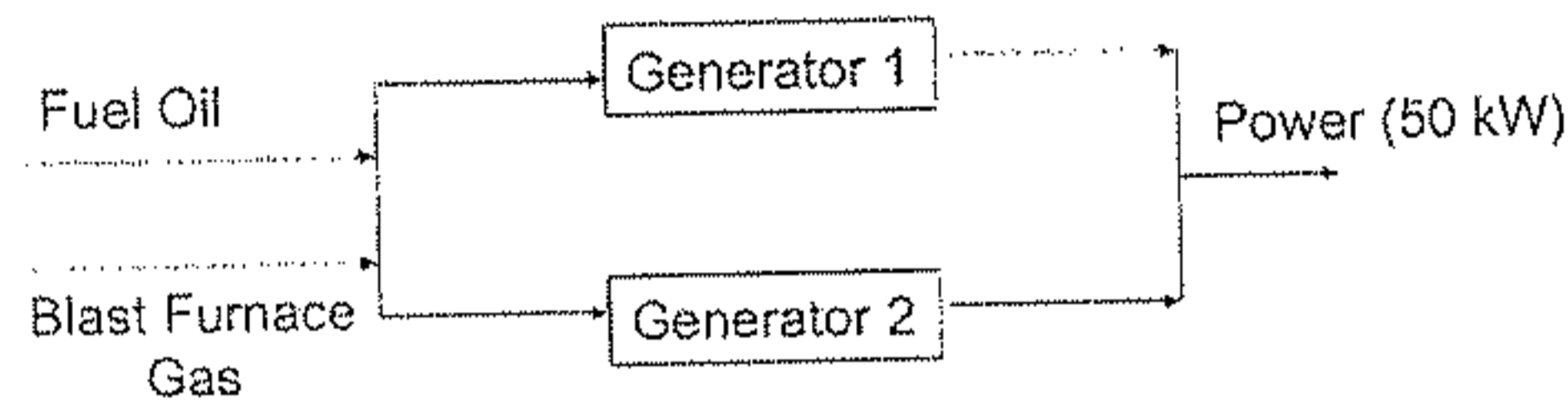


1. A two-boiler turbine-generator combination shown below



is used to produce a power output of 50 MW using a combination of fuel oil or blast furnace gas (BFG). However, to run these generators, BFG is limited to 12 units per hour and additional fuel oil needs to be purchased. To determine the amount of fuel needed, a curve fit has been done so that generator power (x) and fuel required (f) is given by $f = a_0 + a_1x + a_2x^2$. Coefficients for this correlation are given below.

generator	fuel	a_0	a_1	a_2
1	oil	1.4609	0.15186	0.00145
1	gas	1.5742	0.16310	0.001358
2	oil	0.8008	0.20310	0.000916
2	gas	0.7266	0.22560	0.000778

Assume that the generators each produce power between 15 and 35.

- Based on the above information, formulate and solve an optimization problem to minimize the amount of fuel oil purchased using the GAMS solver.
 - Does the above problem satisfy sufficient second order conditions? What is the reduced Hessian?
 - If fuel oil is limited to 10 units per hour and BFG is purchased, What is the minimum BFG for 50 MW?
 - For the above problem, what are the sensitivities of the optimum to a) increasing the power output, b) changing the limits of the generators?
2. Rederive the interior point method developed for $\min f(x), s.t. c(x) = 0, x \geq 0$ to the double-bounded NLP, $\min f(x), s.t. c(x) = 0, x_l \leq x \leq x_u$.
- Extend the derivation of the primal-dual equations for $\min f(x), s.t. c(x) = 0, x \geq 0$ to the double-bounded NLP.
 - Derive the resulting Newton step for these equations and the associate KKT matrix.

Solution - Week 4

a)

Minimize oil purchased.

```

GAMS Rev 233 DII-DII 23.3.3 Mac Intel32/Darwin
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Power Generation via Fuel Oil
C o m p i l a t i o n

53 MODEL FUELOIL /all/;
54 SOLVE FUELOIL USING NLP MINIMIZING OILPUR;
55
56 DISPLAY X.L, P.L, Z.L, OILPUR.L;

COMPILATION TIME = 0.002 SECONDS 3 MB DII233-233 Dec 15, 2009

1 * Define index sets
2
3 SETS G Power Generators /gen1*gen2/
4 F Fuels /oil, gas/
5 K Constants in Fuel Consumption Equations /0*2/;
6
7 * Define and Input the Problem Data
8
9 TABLE A(G,F,K) Coefficients in the fuel consumption equations
10
11
12
13
14 gen1.oil 1.4609 0 .00145
15 gen1.gas 1.5742 .16310 .001358
16 gen2.oil 0.8008 .20310 .000916
17 gen2.gas 0.7266 .22560 .000778;
18
19 PARAMETER PMAX(G) Maximum power outputs of generators /
20 GEN1 35.0, GEN2 35.0/;
21 PARAMETER PMIN(G) Minimum power outputs of generators /
22 GEN1 15.0, GEN2 15.0/;
23 SCALAR GASSUP Maximum supply of BFG in units per h /12.0/
24 PREQ Total power output required in MW /50.0/;
25
26 * Define optimization variables
27 VARIABLES P(G) Total power output of generators in MW
28 X(G, F) Power outputs of generators from specific fuels
29 Z(F) Total Amounts of fuel purchased
30 OILPUR Total amount of fuel oil purchased;
31 POSITIVE VARIABLES P, X, Z;
32
33 * Define Objective Function and Constraints
34 EQUATIONS TPPOWER Required power must be generated
35 PWR(G) Power generated by individual generators
36 OILUSE Amount of oil purchased to be minimized
37 FUELUSE(F) Fuel usage must not exceed purchase;
38 TPPOWER.. SUM(G, P(G)) =G= PREQ;
39 PWR(G).. P(G) =E= SUM(F, X(G,F));
40 FUELUSE(F).. Z(F) =G= SUM((K,G), a(G,F,K)*X(G,F)**(ORD(K)-1));
41 OILUSE.. OILPUR =E= Z("OIL");
42
43 * Impose Bounds and Initialize Optimization Variables
44 * Upper and lower bounds on P from the operating ranges
45 P.UP(G) = PMAX(G);
46 P.LO(G) = PMIN(G);
47 * Upper bound on BFG consumption from GASSUP
48 Z.UP("gas") = GASSUP;
49 * Specify initial values for power outputs
50 P.L(G) = .5*(PMAX(G)+PMIN(G));
51
52 * Define model and solve

```

S O L V E S U M M A R Y

MODEL FUELOIL OBJECTIVE OILPUR
 TYPE NLP DIRECTION MINIMIZE
 SOLVER CONOPT FROM LINE 54

**** SOLVER STATUS 1 Normal Completion
 **** MODEL STATUS 2 Locally Optimal
 **** OBJECTIVE VALUE 3.0403

RESOURCE USAGE, LIMIT 0.033 1000.000
 ITERATION COUNT, LIMIT 10 2000000000
 EVALUATION ERRORS 0 0

C O N O P T 3 version 3.14T
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 Bagsvaerdvej 246 A
 DK-2880 Bagsvaerd, Denmark

Using default options.

The model has 9 variables and 6 constraints
 with 16 Jacobian elements, 4 of which are nonlinear.
 The Hessian of the Lagrangian has 4 elements on the diagonal,
 0 elements below the diagonal, and 4 nonlinear variables.

** Optimal solution. There are no superbasic variables.

CONOPT time total 0.030 seconds
 Of which: Function evaluations 0.001 = 3.7%
 1st Derivative evaluations 0.000 = 1.0%

Workspace = 0.03 Mbytes
 Estimate = 0.03 Mbytes
 Max used = 0.01 Mbytes

	LOWER	LEVEL	UPPER	MARGINAL
----- EQU TPOWER	50.0000	50.0000	+INF	0.1661

TPOWER Required power must be generated

----- EQU PWR Power generated by individual generators

	LOWER	LEVEL	UPPER	MARGINAL
LOWER				

	LOWER	LEVEL	UPPER	MARGINAL
gen1	.	.	.	-0.1661
gen2	.	.	.	-0.1688

----- EQU OILUSE

OILUSE Amount of oil purchased to be minimized

----- EQU FUELOIL Fuel usage must not exceed purchase

	LOWER	LEVEL	UPPER	MARGINAL
oil	.	.	+INF	1.0000
gas	.	.	+INF	0.6782

----- VAR P Total power output of generators in MW

	LOWER	LEVEL	UPPER	MARGINAL
gen1	15.0000	35.0000	35.0000	
gen2	15.0000	15.0000	35.0000	0.0028

---- VAR X Power outputs of generators from specific fuels

	LOWER	LEVEL	UPPER	MARGINAL
gen1.oil	.	4.8980	+INF	.
gen1.gas	.	30.1020	+INF	0.0343
gen2.oil	.	15.0000	+INF	.
gen2.gas	.			

---- VAR 2 Total Amounts of fuel purchased

	LOWER	LEVEL	UPPER	MARGINAL
oil	.	3.0403	+INF	-0.6782
gas	.	12.0000		

---- VAR OILPUR Total amount of fuel oil purchased

	LOWER	LEVEL	UPPER	MARGINAL
OILPUR		-INF	3.0403	+INF

**** REPORT SUMMARY :
 0 NONOPT
 0 INFEASIBLE
 0 UNBOUNDED
 0 ERRORS

---- 56 VARIABLE X.L Power outputs of generators from specific fuels

	oil	gas
gen1	4.898	30.102
gen2		15.000

---- 56 VARIABLE P.L Total power output of generators in MW

gen1	35.000,	gen2	15.000
------	---------	------	--------

---- 56 VARIABLE 2.L Total Amounts of fuel purchased

oil	3.040,	gas	12.000
-----	--------	-----	--------

---- 56 VARIABLE OILPUR.L = 3.040 Total amount of fuel oil purchased

EXECUTION TIME = 0.001 SECONDS 3 MB DII233-233 Dec 15, 2009

USER: Ignacio E. Grossmann G100114/0001AP-DAR
 Carnegie Mellon University, Dept. of Chemical Engineering DC8143
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**** FILE SUMMARY
 Input /Users/Journal/Desktop/fueloil.gms
 Output /Users/Journal/Desktop/fueloil.lst

b) from CONOPT solver, there are no superfluous variables. Hence SSC is successfully satisfied, and reduced version is of zero dimension (no allowable directions / no null space).

c) minimize gas purchased

GAMS Rev 233 DII-DII 23.3. Mac Intel32/Darwin
 02/07/11 15:49:37 Page 1
 Power Generation via Fuel Oil
 C o m p i l a t i o n

53 MODEL FUELOIL /all/;
 54 SOLVE FUELOIL USING NLP MINIMIZING BFGPUR;
 55
 56 DISPLAY X.L, P.L, Z.L, BFGPUR.L;

COMPILATION TIME = 0.001 SECONDS 3 MB DII233-233 Dec 15, 2009

```

4 *OPTION SOLPRINT = OFF;
5
6
7 * Define index sets
8 SETS G Power Generators /gen1*gen2/
9 F Fuels /oil, gas/
10 K Constants in Fuel Consumption Equations /0*2/;
11
12 * Define and Input the Problem Data
13 TABLE A(G,F,K) Coefficients in the fuel consumption equations
14 0 1 2
15 gen1.oil 1.4609 .15186 .00145
16 gen1.gas 1.5742 .16310 .001358
17 gen2.oil 0.8008 .20310 .000916
18 gen2.gas 0.7266 .22560 .000778;
19 PARAMETER PMAX(G) Maximum power outputs of generators /
20 GEN1 35.0, GEN2 35.0/;
21 PARAMETER PMIN(G) Minimum power outputs of generators /
22 GEN1 15.0, GEN2 15.0/;
23 SCALAR OilUP Maximum supply of BFG in units per h /10.0/
24 PREQ Total power output required in MW /50.0/;
25
26 * Define optimization variables
27 VARIABLES P(G) Total power output of generators in MW
28 X(G, F) Power outputs of generators from specific fuels
29 Z(F) Total Amounts of fuel purchased
30 BFGPUR Total amount of fuel oil purchased;
31 POSITIVE VARIABLES P, X, Z;
32
33 * Define Objective Function and Constraints
34 EQUATIONS TPWR Required power must be generated
35 PWR(G) Power generated by individual generators
36 BFGUSE Amount of oil purchased to be minimized
37 FUELUSE(F) Fuel usage must not exceed purchase;
38 TPWR.. SUM(G, P(G)) =G= PREQ;
39 PWR(G).. P(G) =E= SUM(F, X(G,F));
40 FUELUSE(F).. Z(F) =G= SUM(K,G), a(G,F,K)*X(G,F)**(ORD(K)-1));
41 BFGUSE.. BFGPUR =E= Z("gas");
42
43 * Impose Bounds and Initialize Optimization Variables
44 * Upper and lower bounds on P from the operating ranges
45 P.UP(G) = PMAX(G);
46 P.LO(G) = PMIN(G);
47 * Upper bound on BFG consumption from GASSUP
48 Z.UP("oil") = OilUP;
49 * Specify initial values for power outputs
50 P.L(G) = .5*(PMAX(G)+PMIN(G));
51
52 * Define model and solve

```

GAMS Rev 233 DII-DII 23.3.3 Mac Intel32/Darwin
 02/07/11 15:49:37 Page 5
 Power Generation via Fuel Oil
 Solution Report SOLVE FUELOIL Using NLP From Line 54

S O L V E S U M M A R Y

MODEL FUELOIL OBJECTIVE BFGPUR
 TYPE NLP DIRECTION MINIMIZE
 SOLVER CONOPT FROM LINE 54
 ***** SOLVER STATUS 1 Normal Completion
 ***** MODEL STATUS 2 Locally Optimal
 ***** OBJECTIVE VALUE 4.2541

RESOURCE USAGE, LIMIT 0.019 1000.000
 ITERATION COUNT, LIMIT 7 2000000000
 EVALUATION ERRORS 0 0

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 DK-2880 Bagsvaerd, Denmark

Using default options.

The model has 9 variables and 6 constraints
 with 16 Jacobian elements, 4 of which are nonlinear.
 The Hessian of the Lagrangian has 4 elements on the diagonal,
 0 elements below the diagonal, and 4 nonlinear variables.

** Optimal solution. There are no superbasic variables.

CONOPT time Total 0.003 seconds
 of which: Function evaluations 0.000 = 8.8%
 1st Derivative evaluations 0.000 = 5.9%

Workspace = 0.03 Mbytes
 Estimate = 0.03 Mbytes
 Max used = 0.01 Mbytes

----- EQU TPOWER 50.0000 50.0000 +INF 0.2008
 LOWER LEVEL UPPER MARGINAL

TPOWER Required power must be generated

----- EQU PWR Power generated by individual generators
 LOWER LEVEL UPPER MARGINAL

gen1 . . . -0.1929
 gen2 . . . -0.2008

----- EQU BFGUSE . . . 1.0000
 BFGUSE Amount of oil purchased to be minimized
 ----- EQU FUELUSE Fuel usage must not exceed purchase
 LOWER LEVEL UPPER MARGINAL

oil . . . +INF 0.8708
 gas . . . +INF 1.0000

----- VAR P Total power output of generators in MW
 LOWER LEVEL UPPER MARGINAL
 gen1 15.0000 35.0000 35.0000 -0.0079
 gen2 15.0000 15.0000 35.0000 .

---- VAR X Power outputs of generators from specific fuels

	LOWER LEVEL	UPPER LEVEL	MARGINAL
gen1.oil	.	24.0265	+INF
gen1.gas	.	10.9735	+INF
gen2.oil	.	15.0000	+INF
gen2.gas	.	.	+INF

---- VAR Z Total Amounts of fuel purchased

	LOWER LEVEL	UPPER LEVEL	MARGINAL
oil	.	10.0000	-0.8708
gas	.	4.2541	.

---- VAR BFGPUR Total amount of fuel purchased

	LOWER LEVEL	UPPER LEVEL	MARGINAL
BFGPUR	-INF	4.2541	+INF

**** REPORT SUMMARY :

0 NONOPT
 0 INFEASIBLE
 0 UNBOUNDED
 0 ERRORS

---- 56 VARIABLE X.L Power outputs of generators from specific fuels

	oil	gas
gen1	24.026	10.974
gen2	15.000	.

---- 56 VARIABLE P.L Total power output of generators in MW

gen1	35.000,	gen2	15.000
------	---------	------	--------

---- 56 VARIABLE Z.L Total Amounts of fuel purchased

oil	10.000,	gas	4.254
-----	---------	-----	-------

---- 56 VARIABLE BFGPUR.L Total amount of fuel purchased

oil purchased	=	4.254	Total amount of fuel
---------------	---	-------	----------------------

EXECUTION TIME = 0.000 SECONDS 3 MB DII233-233 Dec 15, 2009

USER: Ignacio E. Grossmann
 Carnegie Mellon University, Dept. of Chemical Engineering DC8143
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**** FILE SUMMARY

Input /Users/Journal/Desktop/fueloil.gms
 Output /Users/Journal/Desktop/fueloil.lst

a) Optimal sensitivity

a) total power - marginal value ($v_{gas} = 0.2008$ units for every increase in MW, 0.2008 units of additional gas are required.

b) increasing upper bound on gen1 would

Decreasing lower bound of gen2 would not affect results



2) Interior point method for
Min $f(x)$

s.t. $c(x) = 0$

$x_l \leq x \leq x_u$

KKT conditions

$\nabla f(x) + \nabla c(x)v + u_u - u_l = 0$

$c(x) = 0$

$0 \leq x - x_l \perp u_l \geq 0$

$0 \leq x_u - x \perp u_u \geq 0$

a) primal-dual equations

$\nabla f(x) + \nabla c(x)v + u_u - u_l$

$c(x) = 0$

$(x - x_l) u_l = \mu e$

$(x_u - x) u_u = \mu e$

b) Linearizing primal-dual eqns about x^k, u_l^k, u_u^k

w^k	A^k	I	$-I$	d_x	= -	$\nabla f(x^k) + \nabla c(x^k)v + u_u^k$
$(A^k)^T$				d_v		$c(x^k)$
U_u^k		$(x_u - x^k)$		d_{u_u}		$(x - x^k) u_u^k - \mu e$
U_l^k			$(x^k - x_l)$	d_{u_l}		$(x^k - x_l) u_l^k - \mu e$

we can eliminate d_{u_l} & d_{u_u} to
create a symmetric KKT matrix

$$\begin{bmatrix} W^k + \sum_j \lambda_j + \sum_u \lambda_u & A^k \\ (\Lambda^k)^T & 0 \end{bmatrix} \begin{bmatrix} dx \\ du \end{bmatrix} = \begin{bmatrix} \nabla f_{\mu}(x^k) + \nabla c(x^k) \\ c(x^k) \end{bmatrix} v^k$$

where $\lambda_j = U_j (x^k - x_j)^{-1}$

$\lambda_u = U_u (x^k - x_u)^{-1}$

$$\nabla f_{\mu} = \nabla f(x) - \mu (x^k - x_j)^{-1} e - \mu (x^k - x_u)^{-1} e$$

and $du_u = \mu (x_u - x^k)^{-1} e - (u_u^k + (x_u - x^k)^{-1} U_u^k) dx$

$$du_j = \mu (x^k - x_j)^{-1} e - (u_j^k + (x^k - x_j)^{-1} U_j^k) dx$$